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ABSTRACT

Recommendations for educational change are not focused just on curriculum. Recently a great deal of attention has been focused on reforming the evaluation and assessment of students' mathematical knowledge. Since standardized tests determine what topics will be taught in schools, it seems sensible to refine assessment prior to instituting major curricular change. The National Council of Teachers of Mathematics (NCTM) Curriculum and Evaluation Standards provide a clear picture of how to address mathematical assessment and can direct attention to areas that will provide a clearer picture of student understanding. This publication presents a Framework for Mathematical Tasks reflecting five components from the NCTM Standards: the Focus Areas, Problem Solving, Communication, Connections, and Power and Disposition. The framework is represented by a series of interactive cycles and is used to demonstrate how tasks can be constructed and examined. Appendices contain a student scoring rubric and question sets. (MKR)

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MATHEMATICAL TASKS AND THE NCTM CURRICULUM AND EVALUATION STANDARDS

by

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U.S. students should be first in the world in mathematics and science achievement by the year 2000.
National Education Goal #4

In order for this national goal to be achieved, major changes must take place in the education of our students. Prior to the establishment of this and other national education goals, organizations such as the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), The Mathematical Sciences Education Board (MSEB), and the National Research Council (NRC), just to name a few, were already involved in establishing a plan for change. Numerous publications have been released which not only establish the need for reform in mathematics education, but provide specific direction to this effort. Publications such as The Curriculum and Evaluation Standards (NCTM 89), Everybody Counts - A Report to the Nation on the Future of Mathematics Education (NRC, MSEB 89), Reshaping School Mathematics - A Philosophy and Framework for Curriculum (MSEB 90), Counting on You (MSEB 91), and Professional Standards for Teaching Mathematics (NCTM 91) have brought the need for reform to the forefront of the educational community, and the nation.

The recommendations for change are not focused on just curriculum, but recently a great deal of attention has been focused on evaluation and assessment at the national, regional, state, and local level. Since most curriculum is designed to be tested, and what gets tested gets taught, it only makes sense that we refine assessment prior to instituting major curricular changes. "Textbooks circumscribe what topics may be taught, but tests determine what topics will be taught. Too many of today's standardized tests stray far from both the available and the adopted curriculum; none even gets near the ideal curriculum. Just as new text materials must be developed in parallel with the new curriculum, so also should new strategies of and standards for assessment be developed as the curriculum is defined." (MSEB 90) The NCTM Evaluation Standards provide us with a clear picture of how to address mathematical assessment. Focusing on these standards will allow us to direct our attention to areas which will provide a clearer picture of student understanding. A shift from lower-level, recall assessments to the application of higher-order thinking and reasoning skills is being strongly recommended. "Commonly employed methods of evaluation - especially standardized, paper-and-pencil, multiple-choice tests of 'basic skills' - are themselves obstacles to the teaching of problem-solving and higher-order thinking skills, as well as to the use of calculators and computers." (MSEB 91)

The vision of mathematics education in the *Standards* places new demands on instruction and forces us to reassess the manner and methods by which we chart our students' progress. In an instructional environment that demands a deeper understanding of mathematics, testing instruments that call for only the identification of single correct responses no longer suffice. Instead, our instruments must reflect the scope and intent of our instructional program to have students solve problems, reason, and communicate. Furthermore, the instruments must enable the teacher to understand students' perceptions of mathematical ideas and processes and their ability to function in a mathematical context. At the same time, they must be sensitive enough to help teachers identify individual areas of difficulty in order to improve instruction. (NCTM 89)

The Professional Standards for Teaching Mathematics (NCTM 91) provides us with four clear arenas to address the need for change in our approach to curriculum instruction: tasks, discourse, environment, and analysis. Mathematical tasks can provide us with the type of assessment in line with the NCTM Standards, whether we call them Mathematical Tasks, Authentic Assessment or Authentic Tasks. The following excerpt from the Professional Standards for Teaching Mathematics (NCTM 91) does a very good job of establishing the need for tasks of this nature.

TASKS

> The mathematics tasks in which students engage - projects, problems, constructions, applications, exercises, and so on - and the materials with which they work frame and focus students' opportunities for learning mathematics in school. Tasks provide the stimulus for students to think about particular concepts and procedures, their connections with other mathematical ideas, and their applications to real-world contexts. Good tasks can help students to develop skills in the context of their usefulness. Tasks also convey messages about what mathematics is and what doing mathematics entails. Tasks that require students to reason and to communicate mathematically are more likely to promote their ability to solve problems and to make connections. Such tasks can illuminate mathematics as an intriguing and worthwhile domain of inquiry. A central responsibility of teachers is to select and develop worthwhile tasks and materials that create opportunities for students to develop these kinds of mathematical understandings, competence, interests, and dispositions.

A Framework for Authentic Tasks was developed at the Mid-continent Regional Educational Laboratory (McREL), Aurora, CO which identifies 8 characteristics common to all Authentic Tasks. They are:

1. The task requires concepts, generalizations and processes that are considered critical to specific content areas. (This could be viewed as parallel to the NCTM Curriculum Standards.)

2. The task requires one or more complex reasoning processes including: comparison, classification, structural analysis, supported induction, supported deduction, error analysis, constructing support, extending, decision making, investigation, systems analysis, problem solving, experimental inquiry, and invention. (This could be viewed as parallel to the NCTM Evaluation Standards.)

3. The task requires students to gather information in a variety of ways (e.g., reading, interviewing, making observations, using computerized data bases) and from a variety of sources, some of which are primary sources. (This could be viewed, in part, as parallel to the NCTM Communication Standard.)

4. The task allows for multiple and varied products such as oral reports, panel discussions, video-taped documentaries, and dramatic presentations. (This could be viewed, in part, as parallel to the NCTM Communication Standard.)

5. The task is designed to provide a maximum amount of student control and regulation. (This could be viewed as parallel to the NCTM Power/Disposition Standard.)

6. The task is highly amenable to cooperative/collaborative work. (This could be viewed, in part, as parallel to the NCTM Communication Standard.)

7. The task focuses on issues that are relevant to the community and to the student. (This could be viewed as parallel to the NCTM Connections Standard.)

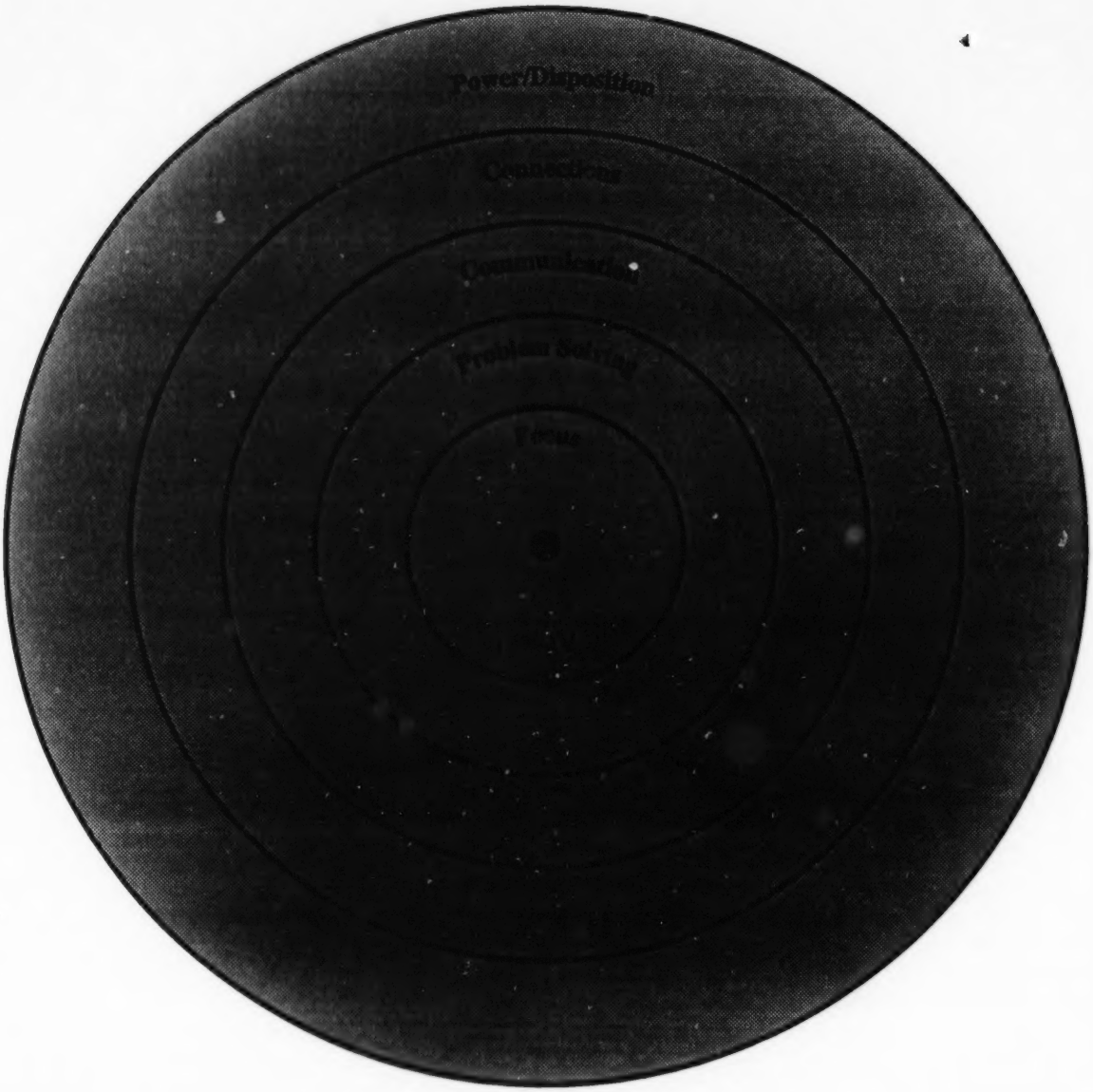
8. The task is long term in nature, reflecting the depth of subject exploration and the use of higher-order thinking skills.

The McREL characteristics interface very nicely with the standard on Mathematical Tasks from the Professional Standards for Teaching Mathematics (NCTM 91). While only one McREL Characteristic has been mapped onto some of the aspects of the NCTM Mathematical Tasks, it should be pointed out that in many instances, more than one of the McREL Characteristics easily aligns with the NCTM Mathematical Tasks. While Characteristic 8 does not, on its own, align with a specific aspect of the Mathematical Task Standard, it can be viewed as aligning with all aspects of Mathematical Tasks through the integration of higher-order thinking skills. Let's look at the outline of Worthwhile Mathematical Tasks to see how it aligns with the Authentic Task Characteristics.

STANDARD 1: WORTHWHILE MATHEMATICAL TASKS	**McREL AUTHENTIC TASK CHARACTERISTICS**
The teacher of mathematics should pose tasks that are based on -	
• sound and significant mathematics;	Characteristic 1. The task requires concepts, generalizations, and processes that are considered critical to specific content areas.
• knowledge of students' understandings, interests, and experiences;	Characteristic 7. The task focuses on issues that are relevant to the community and to the student.
• knowledge of the range of ways that diverse students learn mathematics;	Characteristic 3. The task requires students to gather information in a variety of ways (e.g., reading, interviewing, making observations, using computerized data bases) and from a variety of sources, some of which are primary sources. Characteristic 4. The task allows for multiple and varied products such as oral reports, panel discussions, video-taped documentaries, and dramatic presentations.
• engage students' intellect;	Characteristic 5. The task is designed to provide a maximum amount of student control and regulation.
• develop students' mathematical understandings and skills;	Characteristic 1. The task requires concepts, generalizations, and processes that are considered critical to specific content areas.
• stimulate students to make connections and develop a coherent framework for mathematical ideas;	Characteristic 3. The task requires students to gather information in a variety of ways (e.g., reading, interviewing, making observations, using computerized data bases) and from a variety of sources, some of which are primary sources.

• call for problem formulation, problem solving, and mathematical reasoning;	Characteristic 2. The task requires one or more complex reasoning processes including: comparison, classification, structural analysis, supported induction, supported deduction, error analysis, constructing support, extending, decision making, investigation, systems analysis, problem solving, experimental inquiry, and invention.
• promote communication about mathematics;	Characteristic 4. The task allows for multiple and varied products such as oral reports, panel discussions, video-taped documentaries, and dramatic presentations. Characteristic 6. The task is highly amenable to cooperative/collaborative work.
• display sensitivity to, and draw on, students' diverse background experiences and dispositions;	Characteristic 7. The task focuses on issues that are relevant to the community and to the student.
• promote the development of all students' disposition to do mathematics.	Characteristic 4. The task allows for multiple and varied products such as oral reports, panel discussions, video-taped documentaries, and dramatic presentations. Characteristic 6. The task is highly amenable to cooperative/collaborative work. Characteristic 7. The task focuses on issues that are relevant to the community and to the student.

The reason the McREL Authentic Task Characteristics are mentioned here is to point out that whether you are constructing a task from scratch or examining a task that already exists, you should have the characteristics of a task in mind. These characteristics, while not directly referred to in The Framework for Mathematical Tasks that follows, are important if the mathematical tasks are to truly be "worthwhile". The Framework for Mathematical Tasks illustrated on the next page is comprised of five distinct components: the Focus Areas (NCTM Curriculum Standards); the Evaluation Standard of Problem Solving, in which is subsumed the thinking processes; the Evaluation Standard of Communication; the Evaluation Standard of Connections; and the combined Evaluation Standards of Power and Disposition, represented by a series of interactive circles. To help clarify these components, and to demonstrate how tasks can be constructed and examined, categorical definitions elaborating on the NCTM Curriculum and Evaluation Standards are presented following the illustration.



Framework for Mathematical Tasks

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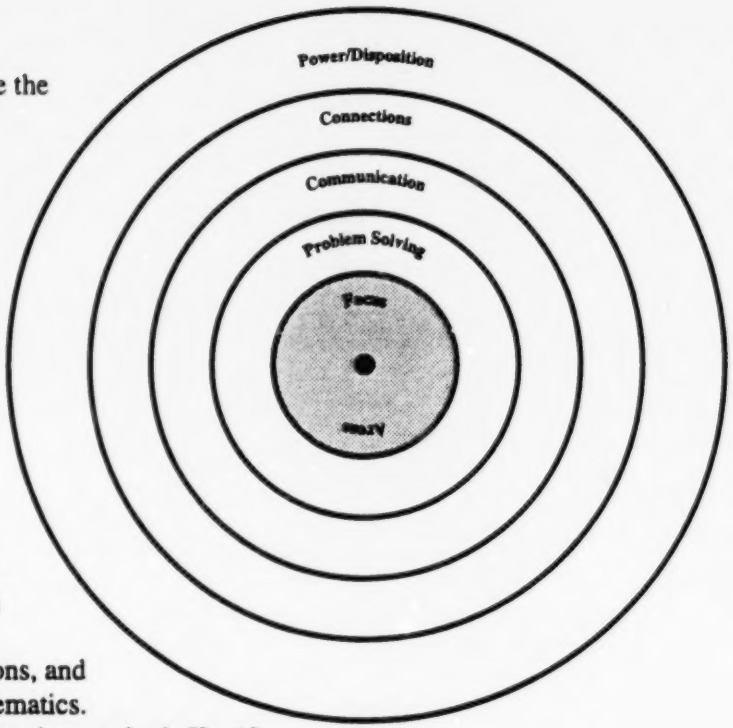
FOCUS AREAS



The NCTM Curriculum Standards are the specific critical concepts, generalizations, and processes they apply to the task. They are recommended areas in which certain mathematical goals can be attained to bring about an increase in students' understanding and application of mathematics. These standards are intended to bring about a change in the content of what students learn as well as the way it is learned.

Reviewing the Curriculum Standards focusing on mathematics provided an opportunity to identify focus areas addressing the concepts, generalizations, and processes considered critical to mathematics. These focus areas are intended to cover the standards K - 12, with some certainly being applicable across all grades (i.e., Data Analysis and Probability), while others are grade group specific (i.e., Algebra and Functions - 5 - 12). These focus areas are intended to cover a number of grade specific standards under a K-12 Focus.

Number Operations - Incorporates the NCTM K-4 Curriculum Standards **Estimation**, **Concepts of Whole Number Operations**, **Whole Number Computation**, and **Fractions and Decimals**, with the 5-8 Curriculum Standard **Computation and Estimation**.

Geometry and Spatial Sense - Incorporates the NCTM K-4 Curriculum Standards **Geometry and Spatial Sense** and **Measurement**, the 5-8 Standards **Geometry** and **Measurement**, and the 9-12 Standards of **Geometry** from both a **Synthetic** and **Algebraic** Perspective.

Patterns and Relationships - Incorporates the NCTM K-4 Curriculum Standard **Patterns and Relationships**, the 5 - 8 Standards **Number and Number Relationships**, **Patterns and Functions**, and **Algebra**, and the 9-12 Standards **Algebra** and **Functions.**

Data Analysis and Probability - Incorporates the NCTM K-4 Curriculum Standard **Statistics and Probability**, the 5-8 Standards **Statistics** and **Probability**, and the 9-12 Standards **Statistics** and **Probability.**

Number Representations - Incorporates the NCTM K-4 Curriculum Standard **Number Sense and Numeration**, with the 5-8 Standards **Number Systems and Number Theory** and **Number and Number Relationships**, and the 9-12 Standard **Mathematical Structure**.

Because of the interrelatedness of these standards, it should be pointed out that while the focus areas are reduced in number from the actual Curriculum Standards as a whole, this reduction does not diminish the scope of the concepts, generalizations and processes considered critical to mathematics. What follows is a synopsis of the Standards as they are identified and presented in the NCTM Curriculum and Evaluation Standards, by Focus Area.

NUMBER OPERATIONS

The mathematics curriculum should develop the estimation and computational concepts of addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals so students can -

- use models to: develop proficiency with basic facts and algorithms; relate fractions to decimals; find equivalent fractions; and explore operations of whole numbers, fractions, and decimals;
- apply whole number, fraction, decimal, integer, and rational number operations to a variety of problem situations;
- recognize that a wide variety of problem structures can be represented by a single operation;
- develop number sense and operation sense as they apply to whole numbers and the concepts of fractions, mixed numbers, and decimals;
- explore, recognize, and use a variety of mental computation and estimation techniques;
- use calculators in appropriate computational situations;
- develop, analyze, and explain methods for solving proportions;
- select and use appropriate computation techniques and determine reasonableness of results.

GEOMETRY AND SPATIAL SENSE

The mathematics curriculum should include two- and three-dimensional geometry so students can-

- describe, model, draw, compare, and classify geometric figures;
- investigate and predict the results of combining, subdividing, changing, and transforming geometric figures;
- develop spatial sense;
- represent and solve problems using geometric models;
- develop the process of measuring and concepts related to units of measurement;

- make and use estimates and actual measurements in problem and everyday situations;
- develop an appreciation of geometry as a means of describing the physical world;
- using transformations, deduce and analyze properties of figures as well as identifying congruent and similar figures.

PATTERNS AND RELATIONSHIPS

The mathematics curriculum should include the study of patterns and relationships as well as algebraic concepts, methods and functions so students can -

- recognize, describe, extend, analyze, and create a wide variety of patterns;
- use patterns and functions to represent and solve problems;
- investigate relationships among fractions, decimals, and percents;
- explore the use of variables and open sentences to express relationships;
- analyze tables, graphs, and functional relationships to identify properties and relationships, explaining how a change in one quantity results in a change in another;
- represent situations that involve variable quantities with expressions, equations, inequalities, and matrices;
- use rules, tables, and graphs as tools to represent and analyze relationships and to interpret expressions, equations, and inequalities;
- operate on expressions and matrices, and solve equations and inequalities;
- investigate inequalities and nonlinear equations, both informally and formally;
- model real-world phenomena with a variety of algebraic methods and functions;
- translate among tabular, symbolic, and graphical representations of functions;
- recognize that a variety of problem situations can be modeled by the same type of function;
- analyze the effects of parameter changes on the graphs of functions.

DATA ANALYSIS AND PROBABILITY

The mathematics curriculum should include experiences with data analysis and probability so students can -

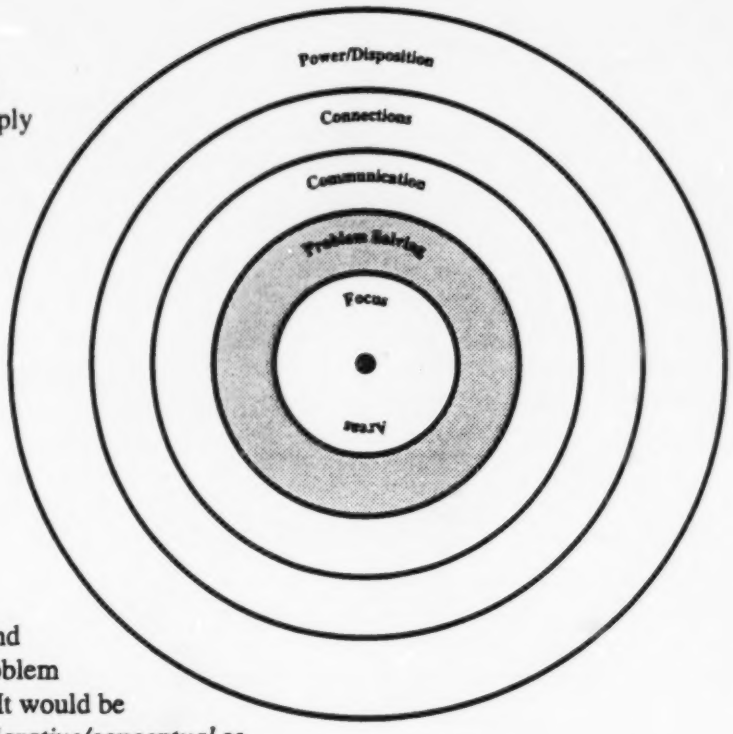
- collect, organize, and describe data;
- construct, read, and interpret displays of data, including tables, charts, and graphs;
- formulate and solve problems that involve collecting and analyzing data;
- explore concepts of chance;
- use experimentation, simulation, and construction of a sample space to determine probabilities;
- make predictions based on experimental or theoretical probabilities;
- make inferences, make convincing arguments, and evaluate arguments based on data analysis.

NUMBER REPRESENTATIONS

The mathematics curriculum should include whole number, fraction, decimal, integer, and rational number concepts and skills as well as measurement so students can -

- construct, understand, and extend number meanings and the attributes of length, capacity, weight, area, volume, time, temperature, and angle through real-world experiences and the use of physical materials;
- understand and apply ratios, proportions, and percents in a wide variety of situations;
- understand our numeration system by relating counting, grouping, and place-value concepts;
- develop number sense for whole numbers, fractions, decimals, integers, and rational numbers;
- interpret, understand, and represent the multiple uses of numbers and equivalence encountered in real world and mathematical problem situations;
- develop the concepts of rates and other derived and indirect measurements;
- develop and apply number theory concepts (e.g., primes, factors, and multiples) in real-world and mathematical problem situations;
- compare and contrast the real number system and its various sub-systems with regard to their structural characteristics.

PROBLEM SOLVING



The Problem Solving and Reasoning Standards can be identified quite simply as the two most powerful areas regarding student capabilities and applicability. These areas allow the student to employ complex reasoning processes in a problem solving content. As a result of the infusion of reasoning in problem solving, the Reasoning Standard will be subsumed in the Problem Solving strand of this model.

Problem Solving and Reasoning, as they are defined in the NCTM Standards, could be viewed more as processes that focus on *procedural* and *declarative/conceptual* aspects of problem solving than as truly separate areas. It would be helpful to define *procedural* and *declarative/conceptual* as they relate to problem solving. *Procedural* processes have a solution that is the result of the execution of a final step. These may be used when the teacher wishes to focus on some aspect of problem solving that involves mastering an ordered sequence of steps. It is this *procedural* aspect of problem solving that most of mathematics seems to gravitate towards. *Declarative/conceptual* processes do not end at a predictable, final step, but rather tend to cease when the properties, attributes, or qualities of the concept or generalization have been exhausted. In some ways, the *declarative/conceptual* processes are more reflective and cause the individual to view the problem with greater depth.

The assessment of students' ability to use mathematics in solving problems and to reason mathematically should provide evidence that they can-

- formulate problems;
- apply a variety of strategies to solve problems;
- solve problems;
- verify and interpret results;
- generalize solutions;
- use inductive reasoning to recognize patterns and form conjectures;

- use deductive reasoning to verify conclusions, judge the validity of arguments, and construct valid arguments;

- use proportional and spatial reasoning to solve problems.

To assist the teacher in looking at problem solving and reasoning in a way that provides greater depth, the following strategies or processes are identified, along with some questions that could help the teacher focus on the mathematical task components. For the most part, these strategies or processes and accompanying questions are divided into two categories, *Procedural* and *Declarative/ Conceptual.*

Procedural

a) Ability to organize information into meaningful sets or categories.

Questions to help focus: Into what categories or groups can these things be organized? What are the rules for category or group membership?

b) Ability to create new processes and expand or alter existing procedures.

Questions to help focus: How can this be improved? What new thing is needed here?

c) Ability to represent a situation in a variety of ways including:
- tables
- symbols
- Euler/Venn diagrams
- graphs
- physical models

Questions to help focus: How can I display this information so it is easy to interpret? Can the information be represented in another way (graphically, symbolically)? Represent this using some type of _____.

d) Ability to be flexible in exploring mathematical ideas and trying alternative methods in solving problems.

Questions to help focus: Is there another way I could have approached the problem? What problems have I already solved which are similar to this one? How can I adapt what I already know?

e) Ability to identify an array of alternative ways of solving the problem.

Questions to help focus: What is the problem to be addressed? What are some of the different ways you might go about solving this problem? Have I solved similar problems? How can I modify other solutions?

f) Ability to reason and analyze efficiently with a willingness to persevere in mathematical tasks.

Questions to help focus: What would be the best way to approach this? How many steps are involved in this problem? Will a single solution be enough? Is there a better solution?

g) Ability to accurately describe the problem situation including the desired goal or information to be generated and the conditions or constraints inherent in the problem.

Questions to help focus: Can I restate the problem? Can I identify the problem? Do I have enough information to solve? Describe what you are trying to accomplish here and what is in your way or what the "givens" are that you must consider.

h) Ability to detect and correct errors.

Questions to help focus: What's wrong with this problem? What are the specific errors that have been made? How can it be fixed?

Declarative/Conceptual

a) Ability to apply their knowledge and value the application of mathematics to solve problems arising in mathematics and other disciplines.

Questions to help focus: What's the general pattern of information here? Where else does this apply? Describe how _____ applies to _____.

b) Ability to demonstrate confidence in using mathematics to solve problems, to communicate ideas, and to reason.

Questions to help focus: What is the most important information? What are the supporting details? How are the pieces related? What is the best strategy to use in solving this problem? How can I best present my solution(s)?

c) Ability to justify and explain one's reasoning?

Questions to help focus: What is the support for my solution? What are the limitations of this solution? Describe the thinking you went through when you _____. Explain why you _____.

d) Ability to evaluate the effectiveness of one's own problem solving behavior.

Questions to help focus: Describe how well your problem solving strategy or process worked. What were the strong points? What were the weak points? Where else might this solution be applied?

e) Ability to describe how ideas or information are similar and different.

Questions to help focus: How are these things alike? How are they different?

f) Ability to generate definitions/descriptions that include the important or defining characteristics of the information being studied.

Questions to help focus: What are the defining characteristics (definitive)? Why/how did this happen? What would/would have happen/ed if (projective)? What are the most important features of _____?

g) Ability to generate and test hypotheses.

Questions to help focus: What do I observe? What can I predict from my observations? What do you think would happen if _____? How would you test this out?

h) Ability to make and justify deductions.

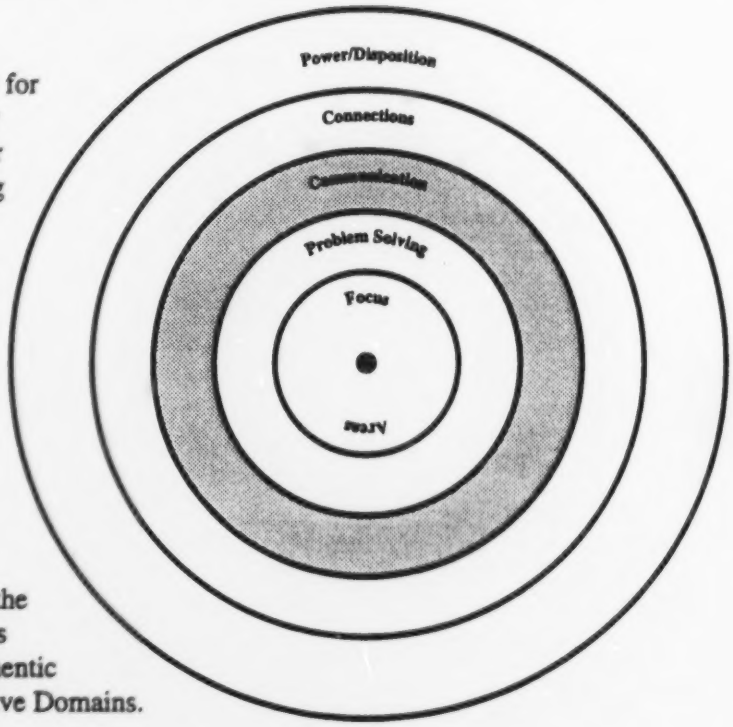
Questions to help focus: Prove that _____. Explain why your method of proof is superior to others. (The best method for this problem.) What has to be true, given the validity of this principle? What is the proof that this must be true?

i) Ability to make and justify inductions.

Questions to help focus: What conclusions can be drawn from your observations? What is the justification for your conclusions? What is the probability for this and what is the support for that conclusion?

COMMUNICATION

For all learners, the principal vehicle for learning is communication - whether performed in conversation with other students, through reading and writing together, or exploring meanings conveyed visually, tactilely, or non-verbally. Communication is in essence the thread in the tapestry of education.



Communication involves an interactive complex of processes. One of the primary functions of development in the communication process is to build confidence in the individual in relation to the goals of the communication process. This applies equally across the Focus Areas, Authentic Task Characteristics, and the Cognitive Domains. These goals are:

1. Use the interactive processes of comprehending and composing to interpret and present understandings through oral, written, visual, tactile, and non-verbal modes.

   Composing - by definition, is the recursive process of conceptualizing and presenting a message which often involves incubation and revision as a message is shaped to fit the intended purpose, audience, and situation.

   Comprehending - by definition, is the creation (or re-creation) of a message by giving attention, perceiving symbols and responding to them, interpreting to shape meaning and attach personal significance and evaluating and applying the derived message.

2. Use language and thinking processes singly or in collaboration with others to construct personally meaningful understandings of concepts and skills.

3. Use the specialized communication processes and skills specific to mathematics, allowing for greater flexibility in regards to the mode of communication.

4. Value the importance of communication particularly as it relates to message formulation/ interpretation, intended audience, feedback as it applies to self-communication and interaction with others.

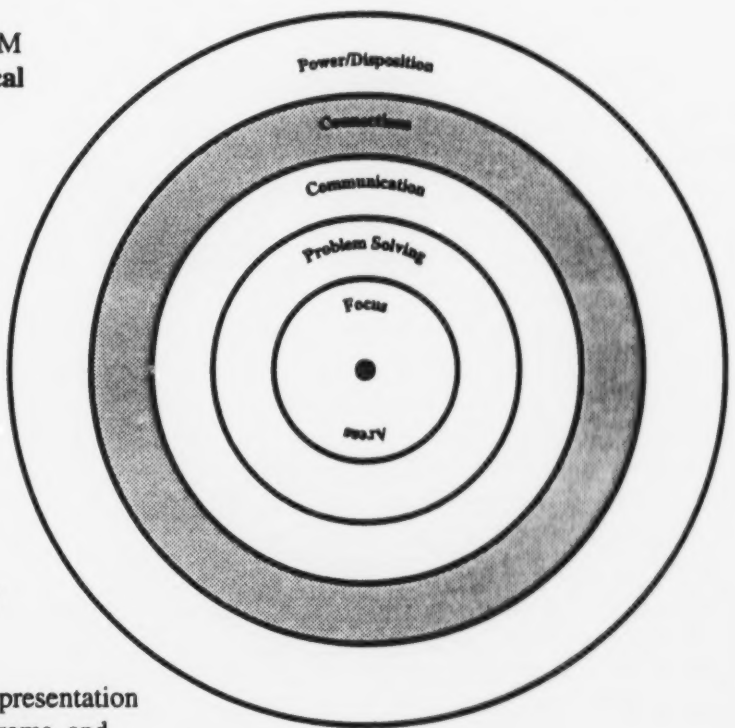
5. Develop the ability to vary communication processes and skills according to function, audience and mode. For the sake of clarification,

Function is defined as the intended purpose(s) of a communication act.
Audience is defined as the intended receiver(s) of a communication act.
Mode is defined as the communication channel(s) used - thought, oral (speaking, listening, and non-verbal expression), written (reading and writing), tactile (actual objects and representations), and visual (graphic and non-alphabetic).

These communication processes and goals manifest themselves specifically in mathematics by allowing students to:

- express mathematical ideas by speaking, writing, demonstrating, and depicting them visually;
- understand, interpret, and evaluate mathematical ideas that are presented in written, oral, or visual forms;
- use mathematical vocabulary, notation, and structure to represent ideas, describe relationships, and model situations.

CONNECTIONS

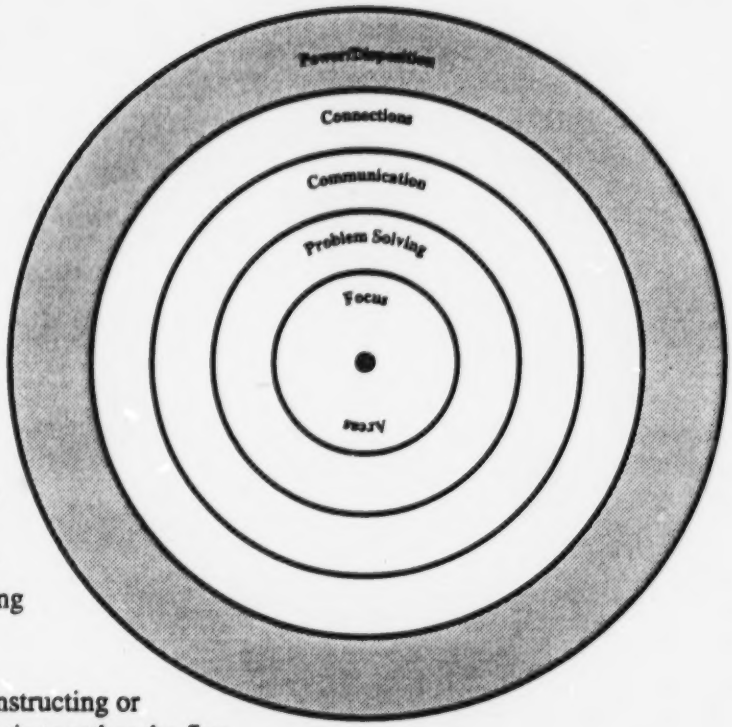


Connections - Incorporates the NCTM Evaluation Standards of **Mathematical Concepts** and **Mathematical Procedures**.

The assessment of students' knowledge and understanding of mathematical concepts and procedures should provide evidence that they can-

- recognize differences in concepts and procedures;
- identify, justify, and execute correct procedures in applications which allows for verification of results;
- translate from one mode of representation to another using models, diagrams, and symbols;
- generate new procedures and extend or modify familiar ones.

MATHEMATICAL POWER and DISPOSITION



Power/Disposition - Incorporates the NCTM Evaluation Standards of **Mathematical Power** and **Mathematical Disposition**.

The assessment of students' mathematical knowledge and disposition should yield information about their-

- knowledge and understanding of concepts and procedures with an inclination to monitor and reflect on their own thinking and performance.

In an effort to outline a process for constructing or examining Mathematical Tasks which tie together the five interactive circles of the Framework, the following steps, along with examples of each step are provided. It should be pointed out that the task used as an example in examining an existing task is a wonderfully rich mathematical task, which incorporates almost every component of the framework. This rich task is used to provide a process which allows the task to be examined in light of each component of the framework.

The task used as an example in constructing a task may appear at times to be somewhat simplified. This simplification is for illustrative purposes only, and is not intended to imply a singular dimension for an mathematical task. In it's ideal state, a worthwhile mathematical task would encompass as many of the facets for each of the five components as possible.

STEPS IN CONSTRUCTING AN AUTHENTIC TASK IN MATHEMATICS

It should be pointed out that the ideal of the framework for mathematics is that a worthwhile mathematical task can be constructed by looking at individual components and developing the task to address each of the components. Conversely, an Authentic Task can also exist, and be examined using the framework to identify the components that the task is addressing. The framework, therefore, exists as a means of development through identification or identification from the task itself. With this in mind, let us begin by viewing the framework as a development process based on the component parts. Some general guidelines to keep in mind:

- All tasks used in the classroom do not have to be authentic tasks. There is still a necessity and place for covering some basic knowledge and skills in a traditional manner.

- All tasks do not have to be assessed. As a matter of fact, mathematical tasks can be used as much for teaching purposes as for assessment. Consequently, many of them are not formally assessed.

Constructing a Task

1. Identify the mathematical content you want the task to use. You can do this by looking at Focus Areas, in which are the Curriculum Standards as identified areas. Within each area are listed some objectives. Ideally, the content should be at a fairly high level, encompassing many mathematical skills and concepts.

Example: I want a task which will incorporate aspects of the Focus Area of Number Operations. In particular, I want to assess a students' **Number Sense** (NCTM Curriculum Standard 6), **Whole Number Computation** (NCTM Curriculum Standard 8 - model, explain, and develop reasonable proficiency with basic facts and algorithms), and **Concepts of Whole Number Operations** (NCTM Curriculum Standard 7 - recognize that a wide variety of problem structures can be represented by a single operation).

2. Identify the aspects of Problems Solving that will be the context of the task. Ideally, you should use the questions to help you determine what the task will contain. You will want the task to incorporate procedural and declarative/conceptual aspects of problem solving.

Example: I want to assess the students' ability to apply a variety of strategies to solve problems and actually solve the problems. I will want them to: identify, justify, and execute correct procedures in applications which will allow for verification of results as well as generating new procedures and extending or modifying familiar ones.

3. Identify the communication focus, from the perspective of the teacher presenting as well as the student producing.

Example: I want the teacher to model the task, utilizing the communication skills necessary to present to the proper audience. Additionally, I want the product to require the students to use written, verbal, and pictorial communication skills.

4. Create a rough draft of a task that incorporates what has been selected in steps 1, 2, and 3.

   Example: I want students to use information from a table to solve a given problem. I want them to add whole numbers to find their solutions.

5. View the task to ensure that it allows for the student to make connections, to their own interest areas as well as across content areas, if possible.

   Example: I want the task to be interesting and relevant to the students and to provide a maximum amount of student control and regulation.

6. Rewrite your rough draft of the task so that it not only allows students to exhibit the identified characteristic(s), but requires it for successful completion of the task.

Example: A fourth grade class was allowed to play games through the afternoon. The teacher said they could use one board game, one card game and one set of puzzles. Table 1 lists what shelves have what type of games and the number of people who can play. If there are 25 students in the fourth grade class, what game selection would insure that everyone could play?

	Shelf #1	Shelf #2	Shelf #3
card game	13	7	9
board game	8	4	5
puzzle set	3	12	8

7. Look at the revised task from the perspective of the students mathematical power, and the ability to demonstrate that to others through the task. It is also important to determine how the students involvement in the task will enhance their disposition toward mathematics.

8. Construct a 3 to 5 point scoring rubric for each of the following:

- the specific Focus Area(s) and objectives you have selected (particular math content)
- the specific Problem Solving processes, procedural and declarative/ conceptual, you have selected (particular complex reasoning processes)
- the Communication aspects you have identified
- the Connections the student can be expected to make
- the aspects of mathematical Power/Disposition the students will develop or demonstrate.

Example (See Appendix 1):

Focus Area: Did the student understand the facts, concepts, and principles associated with Number Sense, Whole Number Computation (model, explain, and develop reasonable proficiency with basic facts and algorithms), and Concepts of Whole Number Operations (recognize that a wide variety of problem structures can be represented by a single operation).

Level 4: The student has mastered essential facts, concepts, and principles and provides new insights into some aspect of the area under study.

Level 3: The student has a good grasp of the facts, concepts and principles in the topic area.

Level 2: The student has an incomplete grasp of the facts, concepts, and principles important for the subject area.

Level 1: The student has a mistaken or marginal understanding of the basic facts, concepts, and principles that are important for the area studied.

Problem Solving: In organizing the information into meaningful sets or categories, did the student use all important components of the process necessary to complete the task?.

Level 4: The student completes all aspects of the process necessary to complete the task. In addition, the student includes items that are not requested, but contribute to the value of the task.

Level 3: Can or has clearly demonstrated all component parts of the process.

Level 2: Adequately completes task but pays inadequate attention to one or more key parts of the process.

Level 1: Excludes critical parts of the process while engaged in task.

Communication: Did the student utilize written, verbal, and visual communication in presenting a solution.

Level 4: The student utilized all three communication processes in presenting a solution.

Level 3: The student utilized two of the identified communication processes in presenting a solution.

Level 2: The student presented a solution utilizing only one communication process.

Level 1: The student did not present a solution.

Connections: Did the student establish relevancy in relation to the task.

Level 4: The student has identified with the task and applied self accordingly.

Level 3: The student has a sense of identification, but lacks in application of self.

Level 2: The student has an incomplete idea of the nature of the task and sees little relevancy.

Level 1: The student has a mistaken or marginal idea of the nature of the task and sees no relevancy.

Power/Disposition: Is the students disposition enhanced as a result of involvement with the task. Can the student apply their mathematical power to the task.

Level 4: The student has applied their mathematical power and benefits from involvement in the task.

Level 3: The student has applied their mathematical power but perceives mathematics as merely the task at hand.

Level 2: The student has benefited by being involved in the task, but has not brought the power of mathematics to this task.

Level 1: The student does not bring the power of mathematics to the task and sees no benefit in the task.

9. Review the task you have constructed. Is this task interesting and relevant enough to engage students and to maintain their involvement? If not, make necessary changes.

10. Present students with the task. Present them with the rubrics as you do so.

11. Have students complete the task.

12. Score the tasks using the rubrics you have created.

Examination of Existing Task

The task used for this example is called "Classroom Buddies" and involves students from one primary classroom to go through a number of problem solving processes to prepare themselves for a meeting with another classroom in a given period (i.e. three weeks). At that meeting, they must pick their classroom buddy (CB) out of the class. Students began the task by compiling a series of questions about their CB, which were delivered to the other classroom (Appendix 2). Once they got their answers back, they compiled a second list of questions, sent them to the classroom, and used the information obtained from the first set of questions to create a small picture of their buddy. This was revised after the second set of answers was received. This process took place once more. One of the questions posed was "How tall are you?", requiring measurement, which took place in a number of ways. Once this information was obtained, actual size drawings were made and displayed in the classroom. Additionally, students created tables, charts, and graphs of the information they collected and displayed them in the room. On the day of the classrooms meeting, the actual sized pictures seemed to come to life as each student successfully identified their "classroom buddy".

1. Identify the mathematical content the task contains by looking at Focus Areas, in which are the Curriculum Standards as identified areas. By going through each Focus Areas objectives, almost like a checklist, objectives which align themselves with the task become readily apparent.

   Example: In this task I see **Number Operations, Geometry and Spatial Sense, Patterns and Relationships, Data Analysis and Probability, and Number Representations**.

2. Identify the aspects of Problems Solving that the task embeds in its context. You should use the questions to help you determine what problem solving aspects the task contains. You will easily identify procedural and declarative/conceptual aspects of problem solving that are contained within the context of the task.

   Example: I can see the students' ability to apply a variety of strategies to solve problems and actually solve the problems. The task requires students to: identify, justify, and execute correct procedures in applications which will allow for verification of results as well as generating new procedures and extending or modifying familiar ones.

3. Look at how the task requires students to communication both in the task itself, and in presenting the information to the appropriate audience.

   Example: The task requires the student to communicate in the final product using written, verbal, and pictorial communication skills.

4. Review the task to identify how it allows for the student to make connections, to their own interest areas as well as across content areas.

   Example: The task appears to be interesting and relevant to the students, since students at this age love to ask questions. The task also provides a maximum amount of student control and regulation, since the questions are theirs and the need to be able to identify their buddy will happen in a few weeks.

5. Look at the task from the perspective of the students mathematical power, and the ability to demonstrate that to others through the task. Does it allow the student to bring their mathematical power to bear on the task. It is important that the students involvement in the task enhances their disposition toward mathematics.

If the task is going to be assessed, it will be necessary to make some type of assessment scoring tool. Tasks lend themselves rather nicely to the use of rubrics.

6. Construct a 3 to 5 point scoring rubric for each of the following:

   - the specific Focus Area(s) and objectives you have identified (particular math content)
   - the specific Problem Solving processes, procedural and declarative/ conceptual, you have identified (particular complex reasoning processes)
   - the Communication aspects you have identified
   - the Connections the student can be expected to make
   - the aspects of mathematical power/disposition the students will develop or demonstrate.

Example (See Appendix 1):

Focus Area: Did the student understand the facts, concepts, and principles associated with Number Sense, Whole Number Computation (model, explain, and develop reasonable proficiency with basic facts and algorithms), and Concepts of Whole Number Operations (recognize that a wide variety of problem structures can be represented by a single operation).

Level 4: The student has mastered essential facts, concepts, and principles and provides new insights into some aspect of the area under study.

Level 3: The student has a good grasp of the facts, concepts and principles in the topic area.

Level 2: The student has an incomplete grasp of the facts, concepts, and principles important for the subject area.

Level 1: The student has a mistaken or marginal understanding of the basic facts, concepts, and principles that are important for the area studied.

Problem Solving: In organizing the information into meaningful sets or categories, did the student use all important components of the process necessary to complete the task?.

Level 4: The student completes all aspects of the process necessary to complete the task. In addition, the student includes items that are not requested, but contribute to the value of the task.

Level 3: Can or has clearly demonstrated all component parts of the process.

Level 2: Adequately completes task but pays inadequate attention to one or more key parts of the process.

Level 1: Excludes critical parts of the process while engaged in task.

Communication: Did the student utilize written, verbal, and visual communication in presenting a solution.

Level 4: The student utilized all three communication processes in presenting a solution.

Level 3: The student utilized two of the identified communication processes in presenting a solution.

Level 2: The student presented a solution utilizing only one communication process.

Level 1: The student did not present a solution.

Connections: Did the student establish relevancy in relation to the task.

Level 4: The student has identified with the task and applied self accordingly.

Level 3: The student has a sense of identification, but lacks in application of self.

Level 2: The student has an incomplete idea of the nature of the task and sees little relevancy.

Level 1: The student has a mistaken or marginal idea of the nature of the task and sees no relevancy.

Power/Disposition: Is the students disposition enhanced as a result of involvement with the task. Can the student apply their mathematical power to the task.

Level 4: The student has applied their mathematical power and benefits from involvement in the task.

Level 3: The student has applied their mathematical power but perceives mathematics as merely the task at hand.

Level 2: The student has benefited by being involved in the task, but has not brought the power of mathematics to this task.

Level 1: The student does not bring the power of mathematics to the task and sees no benefit in the task.

7. Review the task you have constructed. Is this task interesting and relevant enough to engage students and to maintain their involvement? If not, make necessary changes.

8. Present students with the task. Present them with the rubrics as you do so.

9. Have students complete the task.

10. Score the tasks using the rubrics you have created.

REFERENCES

Aurora Public Schools, The Aurora Public Schools Assessment Model, Aurora, CO, 1991.

Iowa Department of Education, A Guide to Developing Communication Across the Curriculum, Des Moines, IA, 1989.

Mathematical Sciences Education Board, Counting on You - Actions Supporting Mathematics Teaching Standards, National Academy Press, Washington, DC 1991.

Mathematical Sciences Education Board and the National Research Council, Reshaping School Mathematics, National Academy Press, Washington, DC, 1990.

Mid-Continent Regional Educational Laboratory, An Overview of McREL's Model for Authentic Assessment, unpublished paper, Aurora, CO, September 1991.

Mid-Continent Regional Educational Laboratory, South Dakota Math Assessment, Practice Task, Grade 4, Aurora, CO, September 1991.

National Council of Teachers of Mathematics, Professional Standards for Teaching Mathematics, Reston, VA, 1991.

National Council of Teachers of Mathematics, Curriculum and Evaluation Standards for School Mathematics, Reston, VA, 1989.

New Jersey State Department of Education, Report of the Eleventh-grade High School Proficiency Test Mathematics Skills Development Committee, Trenton, New Jersey, 1989.

Student Scoring Rubric

	1	2	3	4
Focus Area	The student has a mistaken or marginal understanding of the important basic facts, concepts, and principles.	The student has an incomplete grasp of the facts, concepts, and principles important to subject area.	The students has a good grasp of the facts, concepts and principles in the topic area.	The student has mastered essential facts, concepts, and principles and provides new insights into some aspect of the area under study.
Problem Solving	Excludes critical parts of the process while engaged in task.	Adequately completes task but pays inadequate attention to one or more key parts of the process.	Can or has clearly demonstrated all component parts of the process.	The student completes all aspects of the process necessary to complete the task, including items not requested.
Communication	The student did not present a solution.	The student presented a solution utilizing only one communication process.	The student utilized two of the identified communication processes in presenting a solution.	The student utilized all three identified communication processes in presenting a solution.
Connections	The student has a mistaken or marginal idea of the nature of the task and sees no relevancy.	The student has an incomplete idea of the nature of the task and sees little relevency.	The student has a sense of identification, but lacks in application of self.	The student has identified with the task and applied self accordingly.
Power/ Disposition	The student does not bring the power of mathematics to the task and sees no benefit in the task.	The student has benefited by being involved in the task, but has not brought the power of mathematics to this task.	The student has applied their mathematical power but perceives mathematics as merely the task at hand.	The student has applied their mathematical power and benefits from involvement in the task.

Appendix 1

Question Set #1

What is your name?

Are you a boy or a girl?

What color are your eyes?

What color is your hair?

What color is your skin?

Question Set #2

Is your hair wavy, curly, or straight?

Is your hair to your ears, to your neck, below your shoulders, or below your chest?

How many links tall are you?

Do you have freckles?

How many teeth are you missing?

Question Set #3

What clothes do you like to wear?

What is your favorite color?

What kind of home do you live in?

What is your favorite sport?

Do you have a pet?
If you do, what kind?

Appendix 2